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PROJECT SQUID

TECHNICAL REPORT No. 12

APPROXIMATE THEORY OF COMPRESSIBLE AIR INFLOW THROUGH REED VALVES FOR PULSE JET ENGINES

BY

PAUL TORDA

POLYTECHNIC INSTITUTE
OF BROOKLYN

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PAUL TORDA

POLYTECHNIC INSTITUTE OF BROOKLYN

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APPROXIMATE THEORY OF COMPRESSIBLE AIR INFLOW THROUGH REED VALVES FOR PULSE JET ENGINES

Paul Torda

INTRODUCTION

The purpose of this paper is to present an approximate theory of compressible air inflow between reed valves for pulce jet engines. The exact theory of the air intake process for such engines was developed and presented in two papers published under the sponsorship of Project Squid (References 1 and 2). This exact analysis, however, involves laborious computations and can not, at present, be used for the first assessment of a new design, since, for such a purpose, the number of numerical examples treated would have to be greatly enlarged. The present shorter, even if less exact, analysis is well suited for use in the first stage of a new design. It might well be that the results obtained by the present method are sufficiently accurate so that they can be used for design purposes without reverting to the exact analysis.

In the approximate theory the inertia effects of the fluid are neglected. This neglect introduces an error in the analysis. However, the magnitude of this error may be ascertained by comparing the results obtained by the approximate theory with those obtained by the exact theory.

In the present analysis the shapes of the reeds during movement are determined. This is done by prescribing the forcing function in the differential equation of the forced reed motion and integrating this equation. The inflow, at specific times, is treated as nozzle flow. Velocity, pressure, and density distributions in the fluid are obtained from the nozzle analysis. The method, without alteration, is applicable to the closing period of the valves as well, but the forcing function for this period must be prescribed in agreement with the pressure-time function during combustion. In the present paper, however, for brevity only the opening period of the reeds is treated.

A comparison of the results obtained with those of the exact theory shows good qualitative agreement and also good quantitative agreement in the early part of reed motion. The variations of the results in the latter part of the period are due solely to the following considerations:

1. In the exact analysis, supersonic velocities appeared during inflow, but in the approximate theory velocities above sonic were not admitted.
2. In the numerical example, used for comparison from the exact theory, tapered reeds were treated, whereas in the approximate theory, for the same example, constant moment of inertia was assumed for simplicity.

AIR INFLOW ANALYSIS

For the mathematical treatment of the problem some simplifying assumptions had to be made.

Basic Assumptions. The following basic assumptions have been used in the analysis:

1. The flow upstream of the valves is assumed to be parallel to the valve center planes; i.e., a short cowl of large diameter is assumed ahead of the valve bank, which is built up of a large number of individual valves.
2. The quasi one-dimensional approach employed takes into account the time and space variation of area, flow pressure, and flow velocity between the valves.
3. The analysis considers non-steady, compressible, non-viscous flow between a pair of reed valves with isentropic change of state during inflow.
4. The approximation is introduced in assuming the reeds *frozen* at particular times during the period, at which times the inflow opening is treated as a nozzle. This is equivalent to neglecting the inertia effects of the fluid.
5. Reeds are of constant thickness. This assumption is not necessary, but is introduced for simplification of the numerical computations.

The above assumptions make it possible to determine the pressure, velocity, and density distributions between a pair of reed valves during inflow for a given geometric configuration of the valves and a given or a desired pressure-time function in the combustion chamber.

Analysis. The reeds are considered as cantilever beams of known mass distribution and elastic properties. The same method is applicable to hinged reeds as well, but for brevity, this analysis was not carried out in this paper. If it is postulated that the pressure difference at the free ends of the reeds is to be zero at all times in order to minimize vortex development, then the resultant pressure-force distribution along the reeds may be assumed to be triangular, as shown in Figure 1. Any other distribution of the resultant pressure force may be assumed, as well, without modifying the method presented here. In order to eliminate vortex formation completely, it would be necessary to assure zero velocity difference, as well as zero pressure difference, at the free ends of the reeds. It is not possible at present, however, to introduce this modification into the analysis since no information on the velocity-time function of the gases on the combustion chamber side of the reeds is available.

The magnitude of the pressure difference (Δ_1) at the entrance (clamped end, $x = 0$) is a time variable. In the non-homogeneous fourth order linear differential equation of the reed deflections, the pressure difference function ($\Delta(x,t)$) is the forcing function. This equation can be solved by standard methods (e.g., Ref.3), once the nature of the forcing function is prescribed. The solution of the above equation gives the history of the reed motion, and thus the reed shapes throughout the period are determined. Taking the reed shapes for specified times, one may treat the inflow as a quasi one-dimensional nozzle flow (e.g., Ref. 4). The continuity equation, the Bernoulli equation, and the equation of the change of state of the gas then make it possible to determine the inflow phenomena.

It is possible to postulate either the existence of only sonic and subsonic velocities, or to include also supersonic velocities during inflow. Although in the exact method of solution, Refs. 1 and 2, supersonic velocities do appear, it is believed that because of frictional and other losses, not included in the analysis, no such velocities will occur in actual cases. For this reason, only velocities up to sonic are admitted in the nozzle flow.

Basic Equations and Their Integration. The equation of forced motion of cantilever reeds is:

$$\frac{\partial^2}{\partial\xi^2}\left(EI\,\frac{\partial^2\eta}{\partial\xi^2}\right) + m\frac{\partial^2\eta}{\partial t^2} = f(\xi,t) \tag{1}$$

where EI is the bending stiffness

m is the mass per unit length

ξ and η are the space variables along the reeds and perpendicular to them

t is the time variable

$f(\xi,t)$ is the forcing function.

If m is constant, equation (1) may be written as:

$$EI\frac{\partial^4\eta}{\partial\xi^4} + m\frac{\partial^2\eta}{\partial t^2} = f(\xi,t) \tag{2}$$

If the time variable load per unit length, the forcing function, is given by:

$$f(\xi,t) = r(\xi)\,e^{\omega t} \tag{3}$$

then equation (2) admits a solution of the form

$$\eta(\xi,t) = g(\xi)\,e^{\omega t} \tag{4}$$

and $g(\xi)$ must satisfy the equation

$$EI\,\frac{d^4\eta}{d\xi^4} + m\omega^2\eta = r(\xi) \tag{5}$$

Using the notation

$$\beta^4 = \frac{m\omega^2}{EI} \tag{6}$$

equation (5) becomes

$$\frac{d^4\eta}{d\xi^4} + \beta^4\eta = \frac{1}{EI}\, r(\xi) \tag{7}$$

The solution of the homogeneous equation, corresponding to equation (7), is

$$h(\xi) = e^{\frac{\beta\xi}{\sqrt{2}}}\left[A\cos\left(\frac{\beta\xi}{\sqrt{2}}\right) + B\sin\left(\frac{\beta\xi}{\sqrt{2}}\right)\right] + e^{-\frac{\beta\xi}{\sqrt{2}}}\left[C\cos\left(\frac{\beta\xi}{\sqrt{2}}\right) + D\sin\left(\frac{\beta\xi}{\sqrt{2}}\right)\right] \tag{8}$$

If $r(\xi)$ is given, see Figure 1, by

$$r(\xi) = \frac{\Delta_1}{m\omega^2}\left(1 - \frac{\xi}{H}\right) \tag{9}$$

a particular solution of equation (7) is

$$k(\xi) = \frac{\Delta_1}{m\omega^2}\left(1 - \frac{\xi}{H}\right) \tag{10}$$

and the general solution of equation (7) is

$$g(\xi) = h(\xi) + k(\xi) \tag{11}$$

The boundary conditions to be satisfied by $g(\xi)$ are:

$$\left.\begin{aligned}
&1.\quad \text{For } \xi = 0 \quad g(\xi) = 0\\
&2.\quad \text{For } \xi = 0 \quad \frac{d}{d\xi}[g(\xi)] = 0\\
&3.\quad \text{For } \xi = H \quad \frac{d^2}{d\xi^2}[g(\xi)] = 0\\
&4.\quad \text{For } \xi = H \quad \frac{d^3}{d\xi^3}[g(\xi)] = 0
\end{aligned}\right\} \tag{12}$$

From conditions (12), the four arbitrary constants of integration, A, B, C and D may be determined. They are

$$\left.\begin{aligned}
A &= [C + \lambda_1]\\
B &= 2C - D + \lambda_2\\
C &= -\frac{2e^{-2\alpha}\cos\alpha[\lambda_1\sin\alpha + \lambda_2\cos\alpha] + \lambda_1[1 + e^{-2\alpha}]}{1 + 2e^{-2\alpha} + e^{-4\alpha} + 4e^{-2\alpha}\cos^2\alpha}\\
D &= -\frac{2\lambda_1 - \lambda_2[1 + e^{-2\alpha}] + 2e^{-2\alpha}\sin\alpha\,[\lambda_1\sin\alpha + \lambda_2\cos\alpha]}{1 + 2e^{-2\alpha} + e^{-4\alpha} + 4e^{-2\alpha}\cos^2\alpha}
\end{aligned}\right\} \tag{13}$$

where

$$\lambda_1 = \frac{\Delta_1}{m\omega^2}$$

$$\lambda_2 = \lambda_1[1 + \frac{1}{\alpha}]$$

$$\alpha = \frac{\beta H}{\sqrt{2}}$$

The equations describing the flow in a nozzle and thus between a pair of reeds at any time (t) are:

The continuity equation is

$$u \rho s = u_n \rho_n s_n \tag{14}$$

The Bernoulli equation is

$$u^2 + 2\phi = u_n^2 + 2\phi_n \tag{15}$$

The equation of isentropic change of state is

$$\frac{p}{p_n} = \left(\frac{\rho}{\rho_n}\right)^{\gamma} \tag{16}$$

where

u is the flow velocity

ρ is the fluid density

p is the fluid pressure

s is the cross sectional area between a pair of reeds

γ is the adiabatic constant

$\phi = \int \frac{dp}{\rho} \quad \frac{\gamma}{\gamma-1}\left(\frac{p_0}{\rho_0}\right)\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$ is the enthalpy

subscript zero denotes stagnation conditions

subscript n denotes any specified station along the flow.

If the entrance section $(x = 0)$ is denoted by subscript 1 and if the following non-dimensional variables are introduced

$$U = \frac{u}{u_1} \qquad P = \frac{p}{p_1} \qquad R = \frac{\rho}{\rho_1} \qquad S = \frac{s}{s_1} \tag{17}$$

two equations result for the calculation of the inflow velocities (U) and the inflow pressures (P)

$$\frac{1}{S^2} = \left[\frac{2\gamma}{u_1^2 \, (\gamma - 1)} \frac{p_0}{\rho_0} \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \{1 - P^{\frac{\gamma-1}{\gamma}}\} + 1 \right] P^{\frac{2}{\gamma}} \tag{18}$$

$$U^2 = \frac{2\gamma}{u_1^2 \, (\gamma - 1)} \frac{p_0}{\rho_0} \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \{1 - P^{\frac{\gamma-1}{\gamma}}\} + 1 \tag{19}$$

The relation between u_1 and p_1 is given by

$$u_1^2 = \frac{2\gamma}{\gamma - 1} \frac{p_0}{\rho_0} \{1 - \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \} \tag{20}$$

From measured or desired pressure distribution, the pressure on the combustion chamber side of the reed valves, q, may be given as a function of time at a selected station, say at the entrance $x = 0$. Then a desired pressure variation on the intake side of the reeds p has to be prescribed at the same station $x = 0$ during the intake time. The choice of p may be made from the consideration that, for efficient air intake, $\Delta = p - q$, the pressure difference acting on the reeds should be as large as possible during the opening and as small as possible during the closing time of the valves. The general variation of p and q is drawn in Figure 2. Thus, with the appropriate choice of $p_1 = p_1(t)$, the pressure P and velocity U functions during intake may be determined from equations 18, 19, 20 and the density R variation from equation 16, for the previously determined area ratios S. The transformation from the ξ, η coordinates to the x, y coordinates may be done graphically or analytically.

NUMERICAL EXAMPLE

The numerical example presented has been worked out for a case which has been treated in the exact analysis (Ref. 1, Case 1, area ratio 3:1). The geometric configuration of the valves and the assumed mass distributions are presented in Figure 3. Figures 4, 5, and 6 give the reed deflections, the inflow velocity distributions and the inflow pressure distributions respectively for both the *exact* and the *approximate* theories. Figure 7 shows the pressure distribution on the combustion chamber side of the reeds for the approximate theory.

While comparing the results of the two theories it should be remembered that the different mass distributions assumed influence the instantaneous reed shapes at any corresponding time, as can be seen from Figure 4. The stiffer valves will bend more towards their free end, while the weaker valves more towards their clamped end. The fact should also be recalled that in the exact theory supersonic velocities appeared, but in the approximate theory only velocities up to sonic were admitted.

CONCLUSIONS

Only a qualitative comparison of the *approximate* and *exact* theories could be achieved at this time. The reason for this is that, due to sudden termination of the contract, no time was available for comprehensive numerical computations. From the experience gained during the numerical work for both theories the conclusion can be drawn that the approximate theory will furnish sufficiently accurate determination of the inflow phenomena and thus it may be used for design purposes with confidence.

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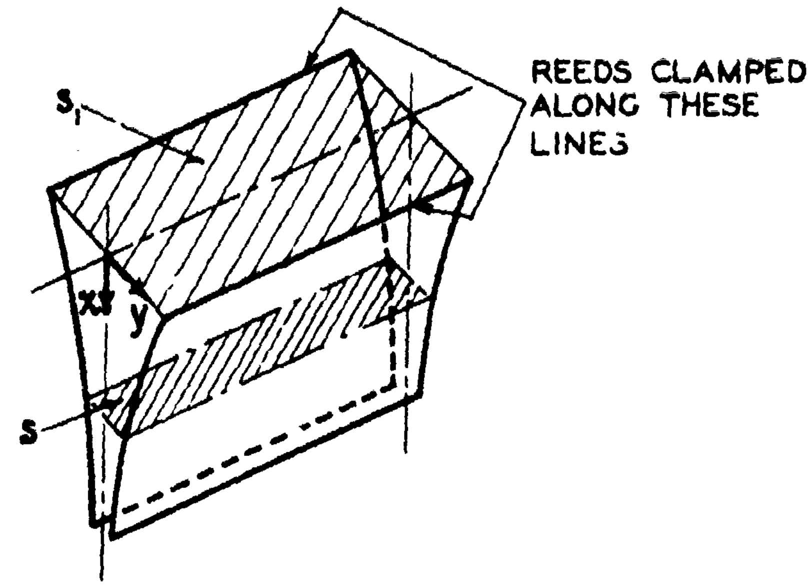

FIGURE 1a

A pair of reeds.

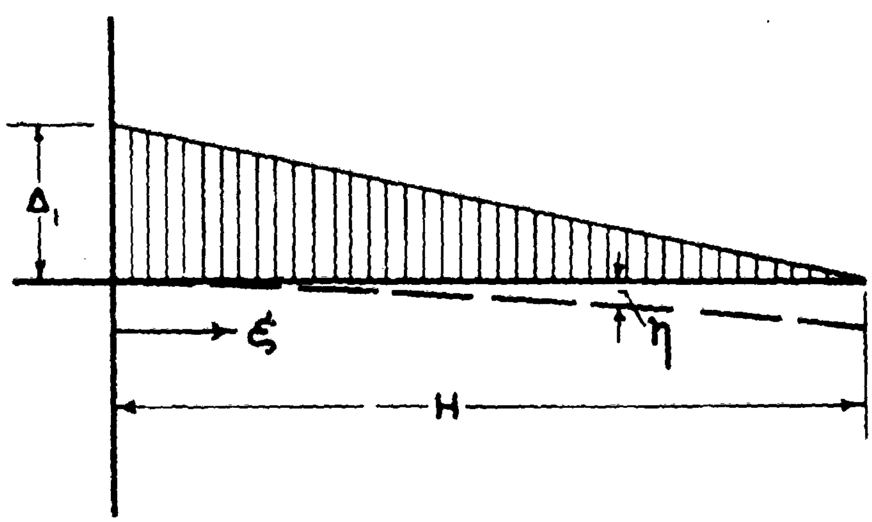

FIGURE 1b

Reed loading.

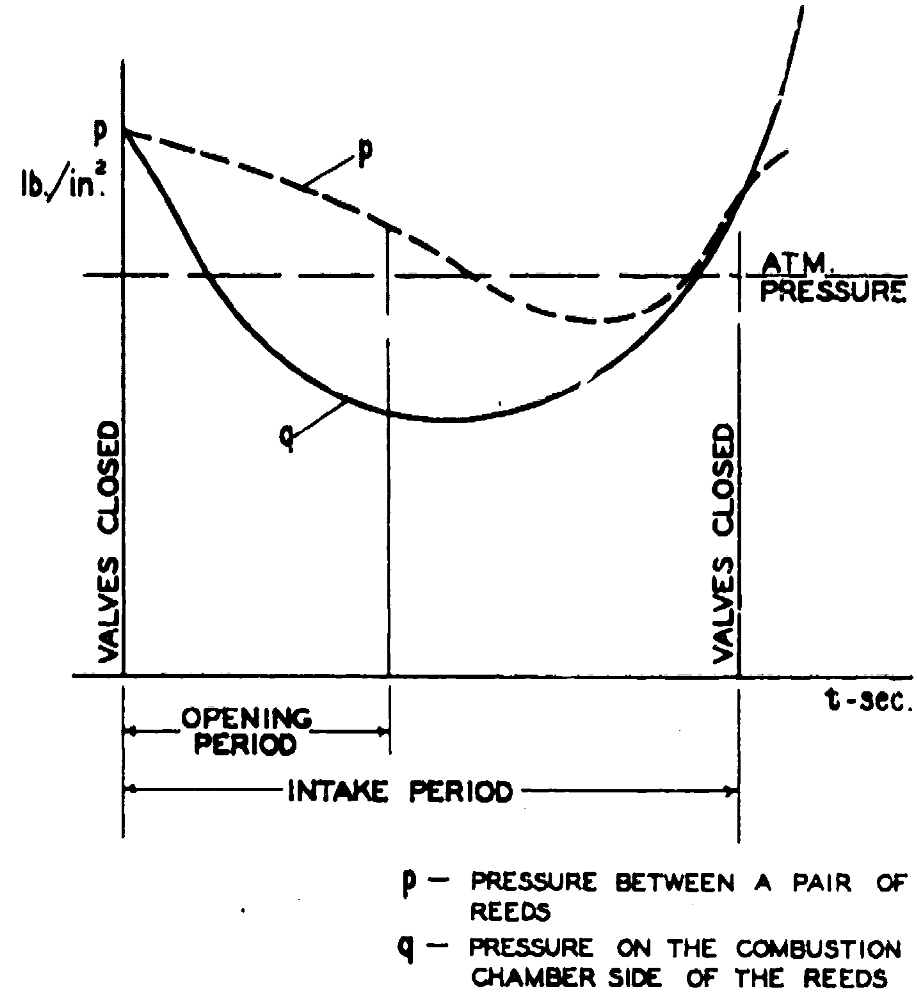

FIGURE 2. Pressure variation on both sides of the reeds at $x = 0$.

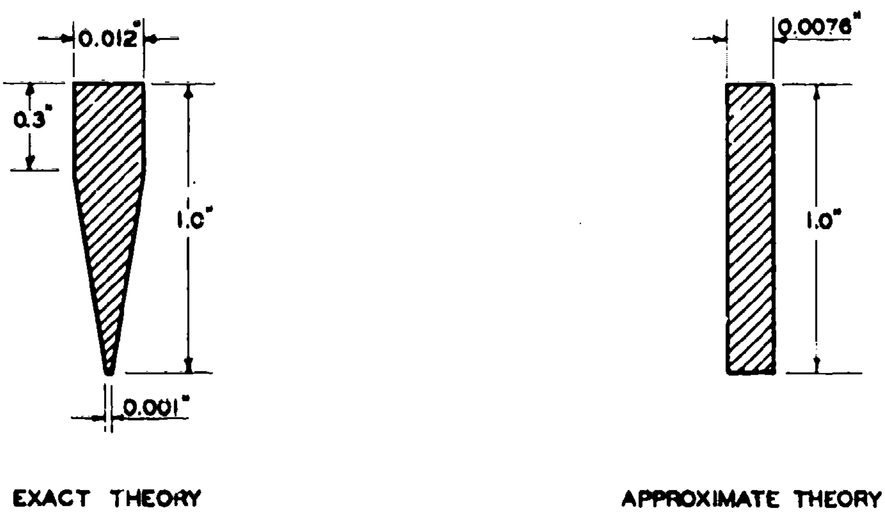

FIGURE 3. Reed dimensions used in numerical examples.

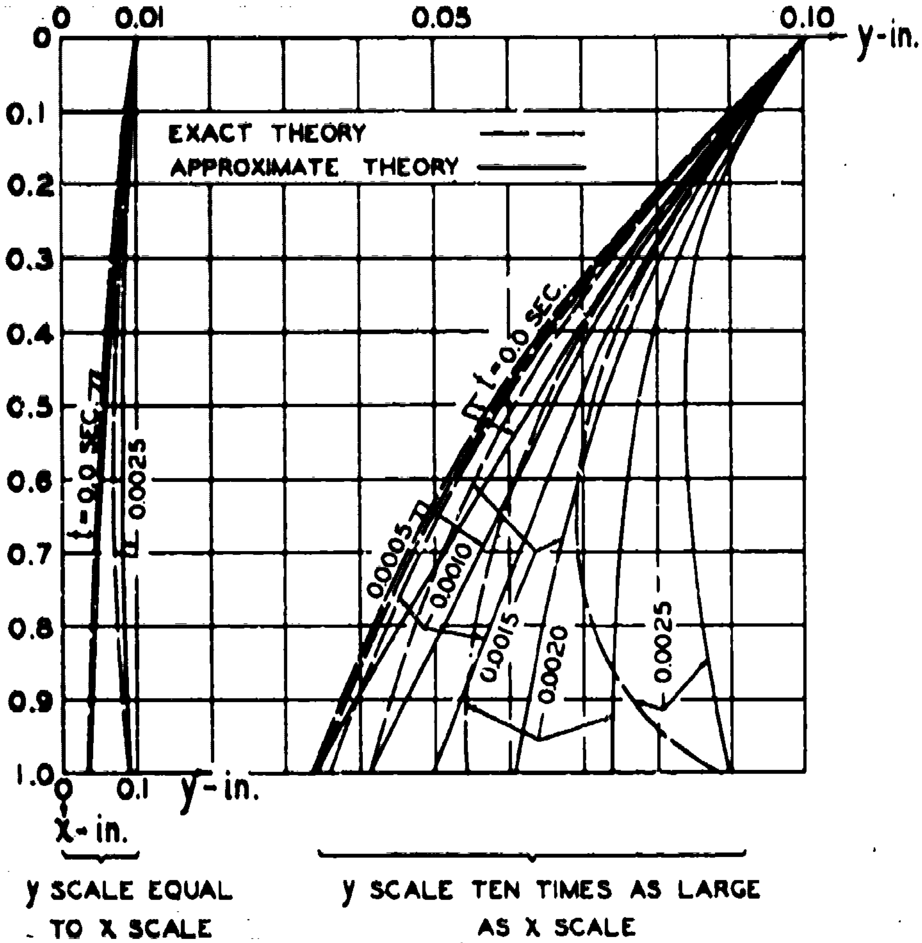

FIGURE 4. Reed deflections

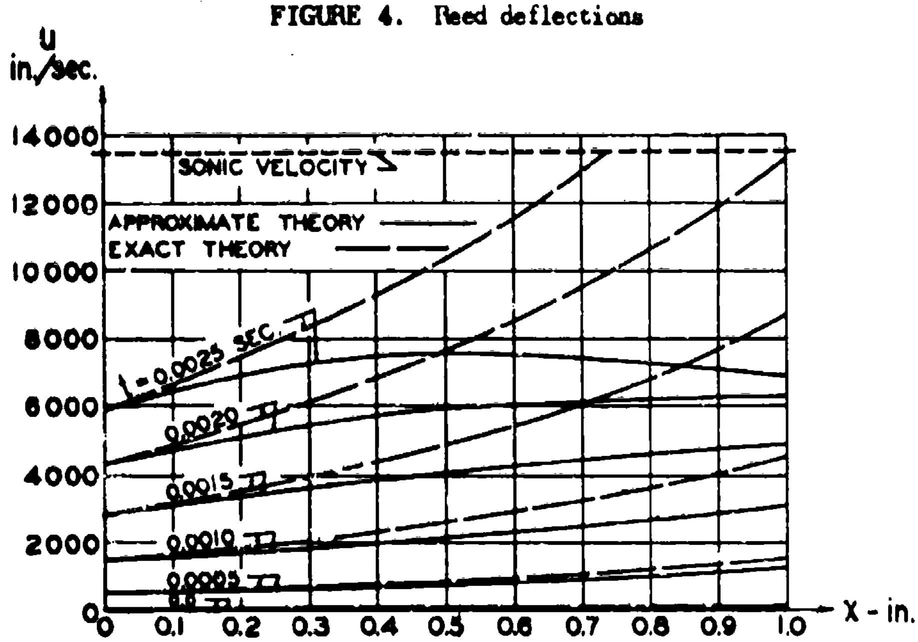

FIGURE 5. Inflow velocity distribution between a pair of reeds.

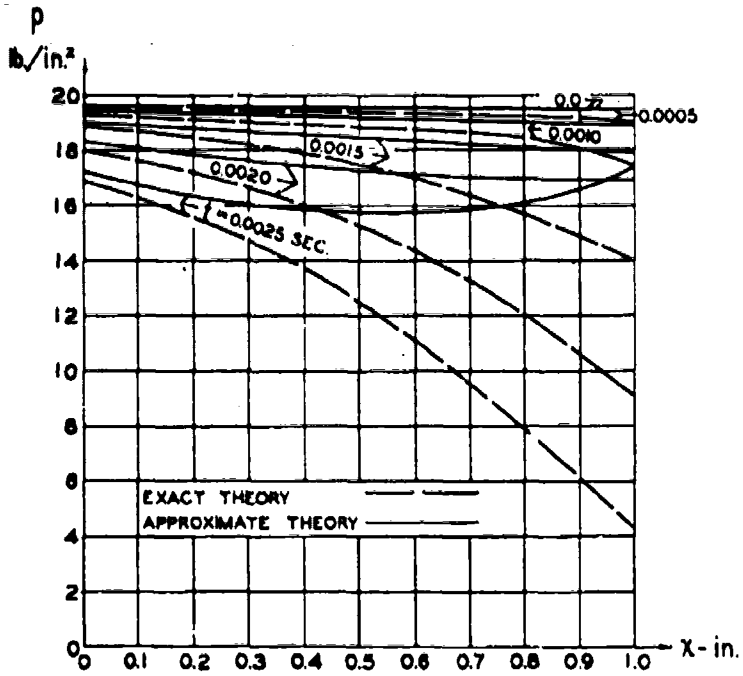

FIGURE 6. Pressure distribution between a pair of reeds.

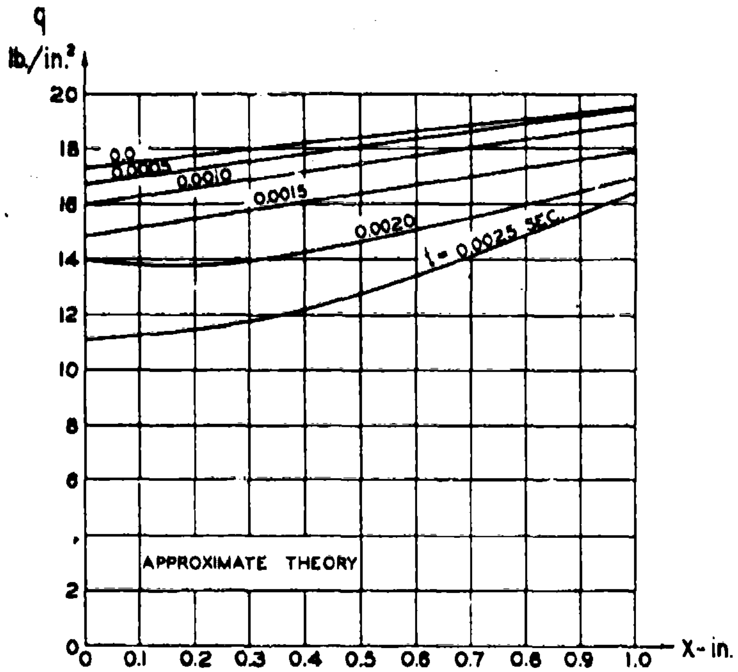

FIGURE 7. Pressure distribution on the combustion chamber side of the reeds.

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